Metamerism

- 2 Colors with different spectral reflectance factors appear the same under one illuminant: \( \text{XYZ}_1 = \text{XYZ}_2 \)
- but different under another illuminant: \( \text{XYZ}_1 \neq \text{XYZ}_2 \)

Metamerism

- Spectrally different but visually identical stimuli (color signals) are called metamers (metameric pairs).
- Illuminant metamerism:
  - two samples may match under one illuminant but not under another.
- Observer metamerism:
  - A set of colors that can be distinguished by the HVS have the same camera code values.
  - A set of colors that cannot be distinguished by the human eye can have different code values.
- If two samples have identical reflectance spectra then they cannot be metameric - they are an unconditional match.
Reproduction Goal

- Colorimetric Reproduction
  - equality of chromaticities and relative luminances
  - Original and reproduction have the same viewing conditions and use illuminants of the same color.
- Spectral Reproduction
  - equality of spectral reflectances or of relative SPD
  - Allows for metameric free reproductions

Multispectral Imaging

- Imaging through more than 3 filters:
  - Multispectral (Hyperspectral):
    - Subsequent exposure through different filters (or filtered illuminants)
    - One exposure onto a sensor with different spectral bands.

The Color Signal

- At the retina the color signal spectra is encoded by three 'numbers': the response of the Long-, Medium-, and Short-wave sensitive cones.

\[
l = \int L(\lambda)C(\lambda) d\lambda \\
m = \int M(\lambda)C(\lambda) d\lambda \\
s = \int S(\lambda)C(\lambda) d\lambda
\]
What is C(\(\lambda\))?
(what is a color signal)

- The color signal is proportional to the product of the spectral power distribution of the light with surface reflectance (assuming matte (non-specular) reflectances)

\[
C(\lambda) \propto E(\lambda)S(\lambda)
\]

What is C(\(\lambda\))?
(what is a color signal)

\[
C(\lambda) = (n\mathbf{l}) E(\lambda)S(\lambda)
\]

(Lambert's Law for Lambertian surfaces)

C(\(\lambda\)) for point source illuminations

- \(n\) denotes surface normal (unit vector)
- \(l\) denotes light direction (unit vector)

\[
C(\lambda) = (n\mathbf{l}) E(\lambda)S(\lambda)
\]

Two point source lights

\[
C(\lambda) = (n\mathbf{l}) E(\lambda)S(\lambda) + (n\mathbf{l}^\prime) E(\lambda)S(\lambda)
\]

\[
C(\lambda) = (n\mathbf{l}) E(\lambda)S(\lambda) + (n\mathbf{l}^\prime) E(\lambda)S(\lambda)
\]

\[
C(\lambda) = (n\mathbf{l} + (n\mathbf{l}^\prime)) E(\lambda)S(\lambda)
\]
N light sources

- The color signal is proportional to the product of light and surface.
- Proportionality is described by a single scalar which depends on the lighting geometry

\[ C(\lambda) = \sum_{i=1}^{N} k_i n_i \Phi(\lambda) S(\lambda) \]

- \( n_i \) is the direction of the \( i \)-th light
- \( k_i \) is the relative power of the \( i \)-th light

\[ C(\lambda) = aE(\lambda)S(\lambda) \]

Infinite light sources

\[ C(\lambda) = \int_{\Psi} n_i k_i E(\lambda) S(\lambda) d\Omega \]

\( \Psi \) denotes the hemisphere around the surface normal
\( \Omega \) denotes a single direction within the hemisphere
\( d\Omega \) denotes an infinitesimal solid-angle

\[ C(\lambda) = aE(\lambda)S(\lambda) \]

The Ambiguity between Light and Surface

\[ C(\lambda) = E(\lambda)S(\lambda) \]

- substitute \( S'(\lambda)=0.5S(\lambda) \) and \( E'(\lambda)=2E(\lambda) \):

\[ C(\lambda) = E'(\lambda)S'(\lambda) = 2E(\lambda)0.5S(\lambda) = E(\lambda)S(\lambda) \]

A 'bright' surface under dim light is indistinguishable from a dim surface under bright light!
Linear Models for Illuminant and Reflectances

• Extend the algebra to incorporate light and surfaces.
• Incorporate linear models of light and surfaces.

Sensor Responses to $m$ Color Signals

\[
[p_a, p_c, p_s] = CR \begin{bmatrix} r & g & b \end{bmatrix}
\]

$3x1$ vector representing the $m$ long-wave sensor responses

$mx1$ vector representing the long-wave sensor sensitivity

$mx3 31x3$ matrix

$3x3$ diagonal matrix

Color Signal

• A color signal is illuminant SPD multiplied by surface reflectance:

\[
C(\lambda) = E(\lambda) \ast S(\lambda)
\]

• Vector multiplication is implemented as diagonal matrix multiplication:

\[
C = SD(e)
\]
Standard Linear Model for Image Formation

\[ P = SD(e)R \]

- **S**(mx31): reflectances
- **D (e)** (31x31): illuminant
- **R**(31x3): sensor sensitivities
- **P**(mx3): sensor response values

Linear models of surface reflectance and illuminants

- Assumption: surface reflectance and illuminants can be approximated with a small number of weighted basis functions:
  \[ S(\lambda) = \sum_{i=1}^{q} S_i(\lambda)\sigma_i, \quad E(\lambda) = \sum_{i=1}^{q} E_i(\lambda)\epsilon_i \]
  - \( S_i \) and \( E_i \) are the basis functions, \( \sigma_i \) and \( \epsilon_i \) are the corresponding weights. \( q \) is the number of basis functions.

Linear models in matrix notation

- Discrete linear model:
  \[ s = B_s\sigma, \quad e = B_e\epsilon \]
  - \( B_s \), \( B_e \): matrix of basis functions (31 x \( q \))
  - \( \sigma \), \( \epsilon \): vector of surface and illuminant weights (\( q \times 1 \))
Representing $m$ surface reflectances

\[ s_i = B_i \sigma_i, \quad s_2 = B_2 \sigma_2, \ldots, \quad s_m = B_m \sigma_m \]

\[ S' = B_s [\sigma_1, \sigma_2, \ldots, \sigma_m] \]

\[ \Omega = [\sigma_1, \sigma_2, \ldots, \sigma_m] \]

\[ S' = B_s \Omega \]

$(B' \text{ is a } 3 \times m \text{ matrix so } S \text{ is } m \times 31)$

Linear models of reflectance and image formation

\[ P = S\Omega R \]

\[ S' = B_s \Omega \]

\[ P = \Omega^T [B_s]^T D(e) R \]

The Lighting matrix

- The lighting matrix for illuminant $E$ is defined as:

\[ \Lambda_E = [B_s]^T D(e) R \]

- Therefore:

\[ P = \Omega^T \Lambda_E \]

(dependent on surfaces)

(dependent on illuminants (and reflectance basis))
Recovering reflectance from RGB

- If \( \mathbf{B} \) is a 31x3 matrix, the reflectance is a linear combination of 3 basis functions.
- For a given illuminant, the lighting matrix is 3x3:
  \[ \Lambda_x = [\mathbf{B}_s]^T \mathcal{D}(\mathbf{e}) \mathbf{R} \]
- The RGBs, \( \mathbf{P} \), are a 3x3 linear transform from the surface weights \( \Omega^T \)
  \[ \mathbf{P} = \Omega^T \Lambda_x \]

Recovering reflectance from RGB

- If reflectance is 3D, the device sensitivities and the illuminant SPD are known, then the surface reflectance can be recovered from the RGBs:
  \[ \mathbf{P} = \Omega^T \Lambda_x \]
  \[ \Omega^T = \mathbf{P}[\Lambda_x]^{-1} \]
  \[ \mathbf{S} = \mathbf{P}[\Lambda_x]^{-1}[\mathbf{B}_s]^T \]

Macbeth Reflectances (24)
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4 Reflectances

DSC Spectral Sensitivities

D55 Illuminant
Lecture 5 - Multispectral Imaging

### Best 3D Approximation

- **rms errors:**
  - rms1 = 0.0336
  - rms2 = 0.0521
  - rms3 = 0.0520
  - rms4 = 0.0248

### Best Reflectance Basis

- Each reflectance is represented as a 31-dimensional vector.
- Surface reflectances are clustered in a small part of 31-dimensional space.

### Best Reflectance Basis

- Representing all reflectances by a single number: mean (31-D vector)

\[ \mu(S) = \frac{1}{m} \sum_{j=1}^{m} s_j \]
Best Reflectance Basis

- Subtracting the mean leaves the “error” (variance) in the data.
- Direction 1 captures a lot of the variance.

Principle Component Analysis

- Eigenvector decomposition of the matrix $S^T S$.
- Direction 1 is the eigenvector corresponding to the largest eigenvalue $d_1$ (largest variance), direction 2 is the eigenvector corresponding to the second largest eigenvalue $d_2$, etc: $d_1 > d_2 > ... > d_31$
- The eigenvectors form the new (orthonormal) basis for reflectances.

Principal Component Analysis

- $S^T$: 31 x m surface matrix
- $\mu_S$: 31 x 1 mean vector
  
  $$S_{\mu}^T = S^T - \mu_S$$

- PCA:
  
  $$S_{\mu}^T S_{\mu} = BDB^T$$

- $B$: eigenvector (basis function) matrix
- $D$: Diagonal matrix of eigenvalues
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Munsell Reflectances (462)

Best 3D Approximation

Basis Functions
### Number of basis functions

**Example:** Only valid for a given illuminant distribution and sensor sensitivities.

<table>
<thead>
<tr>
<th>Number of eigenvectors</th>
<th>Macbeth Colorchecker</th>
<th>Paint Patches 1</th>
<th>Paint Patches 2</th>
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<tr>
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</table>


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### Number of basis functions

<table>
<thead>
<tr>
<th>Number of eigenvectors</th>
<th>Macbeth Colorchecker</th>
<th>Paint Patches 1</th>
<th>Paint Patches 2</th>
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</thead>
<tbody>
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<td>reflectance factor</td>
<td>reflectance factor</td>
<td>reflectance factor</td>
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<td></td>
<td>mean ( \Delta E_{94} (D50, 2') )</td>
<td>rms error</td>
<td>mean ( \Delta E_{94} (D50, 2') )</td>
</tr>
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<td></td>
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<td></td>
<td>mean ( \Delta E_{94} (D50, 2') )</td>
<td>rms error</td>
<td>mean ( \Delta E_{94} (D50, 2') )</td>
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<tr>
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<td>0.002</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Imai and Berns

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### #BF vs. Colorimetric Errors

- Estimation using 3 filters: Mean \( \Delta E = 0.4 \)
- Estimation using 7 filters: Mean \( \Delta E = 3.0 \)

EPFL/CI 2005/Susstrunk

13
Number of Color Filters

- Determined by the desired accuracy of spectral reconstruction.
- Determined by the noise in the imaging system and the quantization.
  - Hardeberg

#BF vs. noise

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## #BF vs. noise

Hardeberg

- ![Graph](image)

## Optimal #BF vs. Quantization

Hardeberg

- ![Graph](image)

## Choice of Color Filters

- Filters need to be chosen so that they (when multiplied with the illuminant and camera characteristics) span the same vector space as the reflectances that are to be acquired in a particular application:

  \[
  \max \left\{ \langle P, D(e)R \rangle = \frac{\text{trace}(NN^T O^O)}{\sigma} \right\}
  \]

  - Vora and Trussell

---

EPFL/CI 2005/Susstrunk
Choice of Color Filters

- In reality, the best filters from a set of physically realizable filters (Wratten, Hoffmann, Schott) is chosen so that for the first filter, the norm of the projection on the reflections basis subspace is maximized.
- Subsequent filters are chosen so that the components orthogonal to the first one are optimized.
  - Hardeberg

Image Formation with Multispectral Camera

- The Image Formation with a multispectral camera can be written as:
  \[ P_y = S D(e) R_y \]

  \( P \) (m x q): response values for m surface reflectances and q sensors
  \( S \) (m x 31): surface reflectances
  \( D(e) \) (31 x 31): illuminant
  \( R \) (31 x q): sensor sensitivities

  \[ P_y = S \Phi_y \]
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Reconstruction

- One method:

\[ S = P_q \Phi_q^+ = P_q (\Phi_q^T \Phi_q)^{-1} \Phi_q^T \]

- Does not work well when q \( \ll \) 31, as it minimizes error in camera space. Also subject to large errors in the presence of noise. Other methods exist (see Hardeberg, 1999).

Failed Reconstruction

\[ \text{rms}=0.3327 \]


Reading Assignment

  - Chapter 10, Multispectral Imaging (see web)
Lecture 5 - Multispectral Imaging

References